



Chettinad

College of Engineering & Technology

Approved by AICTE, New Delhi and Affiliated to Anna University, Chennai

Academic Year 2023 -2024

Notes of Lesson

Year/Semester: I/II

Department : ECE

Unit : I to V

Date: 06.03.2024

Subject Code/Title : EC3251/ Circuit Analysis

Total Hours : 60 Hrs

Faculty Name : Mrs. A. Karthikeyani

Subject Credit : 4

Unit-II Network Theorems and Duality

UNIT - II

Network Theorem and Duality

Useful Circuit Analysis techniques - Linearity and Superposition, Thevenin and Norton Equivalent Circuits, Maximum power Transfer, Delta-Wye Conversion, Duals, Dual Sources Circuits, Analysis using dependent current sources and voltage sources.

Linearity and Superposition principle:

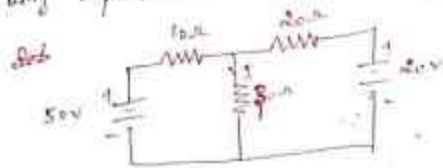
Superposition theorem:

"In any linear and bilateral network or circuit having multiple independent sources, the response of an element will be equal to the algebraic sum of the responses of that element by considering one source at a time".

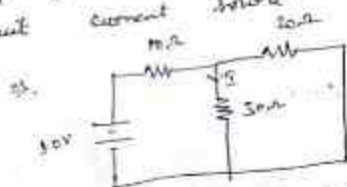
Steps to apply:

- (1) select a single source acting alone. Short the other voltage sources and open the current sources of interest.
- (2) Find the "I" through or the voltage across the required element, due to the source under consideration, using a firm suitable network simplification techniques.
- (3) Repeat the above 2 steps for all the sources.
- (4) Add all the individual effects produced by individual sources, to obtain the total current in or voltage across the element.

② Find the current I for the circuit shown by using Superposition theorem.



Step 1:
 \Rightarrow We select any one of the voltage source (10V) and short circuit the voltage source and open circuit current source.



\Rightarrow Find the R_{eq} :

$$\therefore R_{eq} = 20\Omega$$

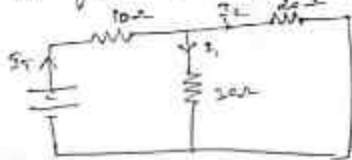
$$\frac{20 \times 30}{20 + 30} = \frac{600}{50} = 12\Omega$$

10Ω series with 12Ω

$$\Rightarrow 10 + 12 = 22\Omega$$

$$\begin{aligned} \Rightarrow \therefore \text{Total current } I_T &= \frac{V_s}{R_T} \\ &= \frac{50}{22} = 2.27\text{A} \end{aligned}$$

\Rightarrow To find I_1 use current division rule.



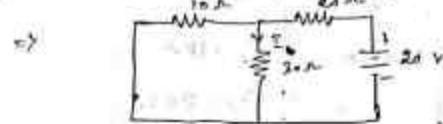
$$I_1 = I_T \times \frac{\text{Resistance of opp. Branch}}{R \text{ of opp. Branch} + R \text{ of current Branch}}$$

$$= 2.27 \times \frac{20}{20 + 30} = 2.27 \times \frac{20}{50}$$

$$\boxed{I_1 = 0.908\text{A}} \downarrow$$

Step 2:

\Rightarrow select 20V source and remove short circuit
 Volt: source and open circuit current source.

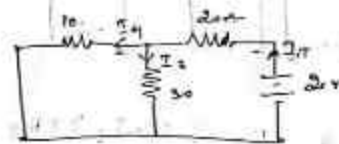


[Same steps will be followed as like step 1]

$$\Rightarrow R_{eq} = 27.5\Omega$$

$$\Rightarrow I_T = \frac{20}{27.5} = 0.727\text{A} \Rightarrow \boxed{I_T = 0.727\text{A}}$$

\Rightarrow To find I_2 use current division rule



$$\Rightarrow I_2 = I_T \times \frac{10}{10 + 30} \Rightarrow \boxed{I_2 = 0.181\text{A}} \downarrow$$

Step 3:

Both I_1 & $I_2 \Rightarrow \downarrow$ [Downward]

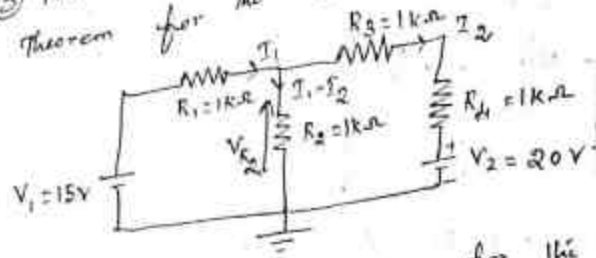
If both directions are same [Downward or Upward]

Add the currents

$$\therefore I_{30\Omega} = 0.908 + 0.181$$

$$\boxed{I_{30} = 1.089\text{A}} \downarrow$$

② Find the voltage across R_2 using superposition theorem for the circuit shown in figure.



Write the mesh matrix for the given circuit:

$$\begin{bmatrix} 2000 & -1000 \\ -1000 & 3000 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 15 \\ -20 \end{bmatrix}$$

Voltage across $R_2 = (I_1 - I_2) \times R_2$

$$\therefore V_1 = 15V, V_2 = 20V$$

$$\Delta = \begin{vmatrix} 2000 & -1000 \\ -1000 & 3000 \end{vmatrix} = 5 \times 10^6$$

$$\Delta_1 = \begin{vmatrix} 15 & -1000 \\ -20 & 3000 \end{vmatrix} = 25 \times 10^3$$

$$\Delta_2 = \begin{vmatrix} 2000 & 15 \\ -1000 & -20 \end{vmatrix} = -25 \times 10^3$$

$$\therefore I_1 = \frac{\Delta_1}{\Delta} = 5 \text{ mA}$$

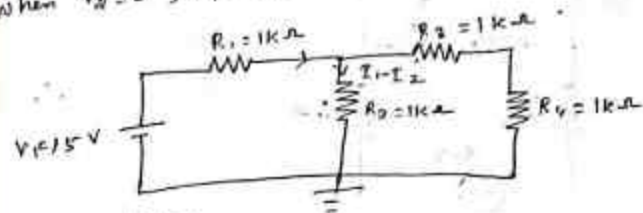
$$I_2 = \frac{\Delta_2}{\Delta} = -5 \text{ mA}$$

$$\therefore I_1 - I_2 = 10 \times 10^{-3} = 10 \text{ mA}$$

$$\therefore V_{R_2} = (I_1 - I_2) \times R_2 = 10 \times 10^{-3} \times 10^3 = 10V$$

Case (i):

When $V_2 = 0$; $V_1 = 15V$



Mesh matrix:

$$\begin{bmatrix} 2000 & -1000 \\ -1000 & 3000 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 15 \\ 0 \end{bmatrix}$$

$$\therefore \frac{\Delta_1}{\Delta} = I_1 = 9 \text{ mA}, I_2 = \frac{\Delta_2}{\Delta} = 3 \text{ mA}$$

$$\therefore I_1 - I_2 = 6 \text{ mA} \Rightarrow V_{R_2} = (I_1 - I_2) \times R_2 = 6 \times 10^{-3} \times 10^3 = 6V$$

Case (ii):

When $V_1 = 0$; $V_2 = 20V$

$$I_1 = -4 \text{ mA}, I_2 = -8 \text{ mA}$$

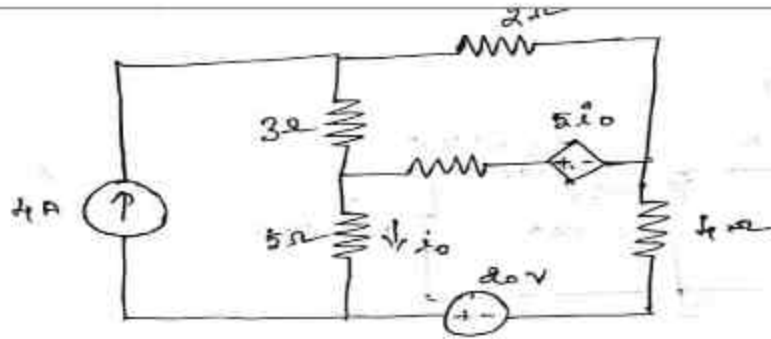
$$\therefore I_1 - I_2 = (-4 + 8) \times 10^{-3} = 4 \text{ mA}$$

$$\therefore I_1 - I_2 = 4 \text{ mA} \Rightarrow V_{R_2} = 4 \times 10^{-3} \times 10^3 = 4V$$

Add Eq. (i) & Eq. (ii)

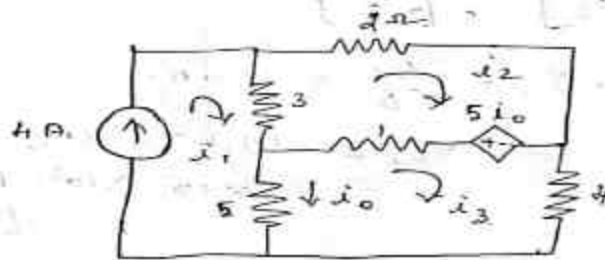
$$\boxed{V_{R_2} = 10V}$$

③ Find I_0 in the circuit in fig using superposition (Dependent source).



Soln:

Mesh analysis:



Loop 1

$$-4 + 8i_1 - 3i_2 - 5i_3 = 0$$

$$\Rightarrow 8i_1 - 3i_2 - 5i_3 = 4$$

Loop 2:

$$2i_2 + i_2 - i_3 + 3i_2 - 3i_1 - 5i_0 = 0$$

$$-3i_1 + 6i_2 - i_3 - 5i_0 = 0$$

↳ (2)

Loop 3:

$$10i_3 - i_2 - 5i_1 + 5i_0 = 0$$

↳ (3)

$$\therefore i_3 = i_1 - i_0 = 4 - i_0$$

$$i_1 = 4 - i_0 \quad ; \quad i_3 = 4 - i_0$$

∴ Substitute (2) & (3)

$$-12 + 6i_2 - 4 + i_0 - 5i_0 = 0$$

$$-16 + 6i_2 - 4i_0 = 0$$

$$6i_2 - 4i_0 = 16$$

$$3i_2 - 2i_0 = 8 \quad \text{--- (4)}$$

$$-40 - 10i_0 - i_2 - 20 + 5i_0 = 0$$

$$-5i_0 - i_2 = -20$$

$$\Rightarrow 5i_0 + i_2 = 20 \quad \text{--- (5)}$$

$$\text{--- (4)} \times 3 \Rightarrow 15x + 3y = 60$$

$$-5x + 4y = 20$$

$$\text{--- (5)} \times 3 \Rightarrow 15x + 3y = 60$$

$$-5x + 4y = 20$$

$$77x = 68 \quad 57$$

$$x = 52/17$$

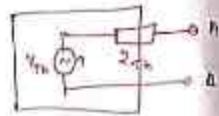
$$i_0 = 3.0588235$$

Thevenin's Theorem

Statement

"Any linear active n/w with o/p terminals A & B as shown in figure can be replaced by a single voltage source ($V_{th} = V_{oc}$) in series with a single impedance ($R_{th} = Z_{in}$)."

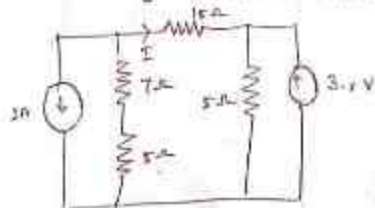
V_{th} → Thevenin volt ; V_{oc} = OC voltage



$R_{th} \equiv Z_{th}$ (if C & J are absent)

$$I_L = \frac{V_{th}}{R_{th} + R_L}$$

⑥ Find 'I' using Thevenin's theorem:



Solution: (we need to find R_{th})

Consider the R_L [I current flowing through R_L]

Step 1: Go find R_{th} :

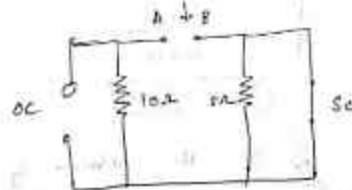
In order to find R_{th} we need to do 3 steps

(i) Remove the R_L .

(ii) SC the voltage source (i.e. OC the current source)

⇨⇨⇨

The circuit diagram will be,



We need to find Total 'R' across A & B.

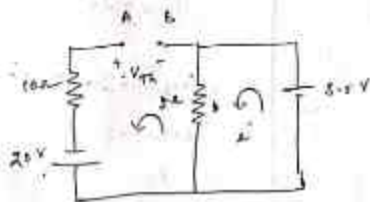
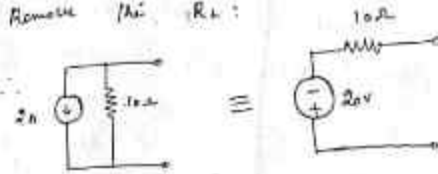
Note: If SC or in parallel with 5Ω, don't consider

that 'R' (5Ω)

$$R_{th} = 10\Omega$$

Step 2: Go find V_{th} :

Remove the R_L :



$$[\because I = \frac{3.5}{5}]$$

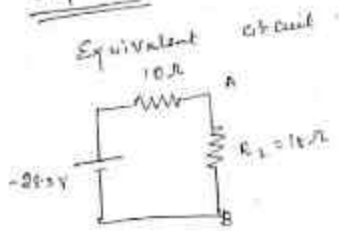
Apply the KVL to the loop which is having V_{th} :

→ A & B terminals is opened current across 10Ω,

$$5 \times 0 + V_{th} + 20 = 0 \quad (\text{or}) \quad -5 \times 0.7 - V_{th} - 20 = 0$$

$$\Rightarrow V_{th} = -23.5V$$

Step 3:

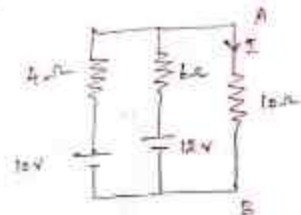


$$I = \frac{V_{th}}{R_{th} + R_L}$$

$$= \frac{-28}{10 + 16}$$

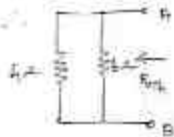
$$I = -0.94A$$

- (P) Determine the current I in the network by
 (Q) using Thevenin's theorem:



Step 1: To find R_{th} :

- (i) Remove the R_L [$\rightarrow I \Rightarrow R_L = 10\Omega$]
 (ii) Remove voltage source by short



$$R_{th} = \frac{4 \times 6}{4 + 6} = \frac{24}{10} = 2.4\Omega$$

$$R_{th} = 2.4\Omega$$

Step 2: To find V_{th} :



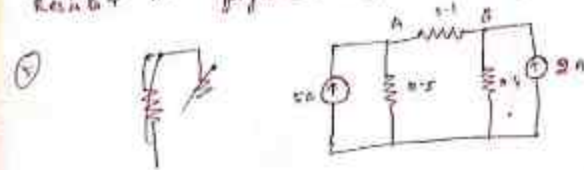
Step 4:

$$I_L = \frac{V_{th}}{R_{th} + R_L}$$

$$= \frac{10}{2.4 + 10}$$

$$= 0.821A$$

- (4) It is required to find current through the 10Ω resistor in figure using Thevenin's theorem.

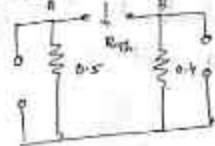


Solution:

- (i) To find R_{th}

(1) Here $R_L = 10\Omega$ (Disconnect)

(2) Remove current source by ∞



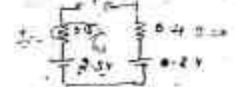
$$R_{th} = 0.5 + 0.6 = 0.9\Omega$$

- (ii) To find V_{th} :

Remove R_L & apply KVL for the V_{th} loop



$$[V_{th} = V_{AB}]$$



$$\rightarrow V_{th} + 0.5 \times (5) - 0.4 \times 2 = 0 \quad [A = +ve, B = -ve]$$

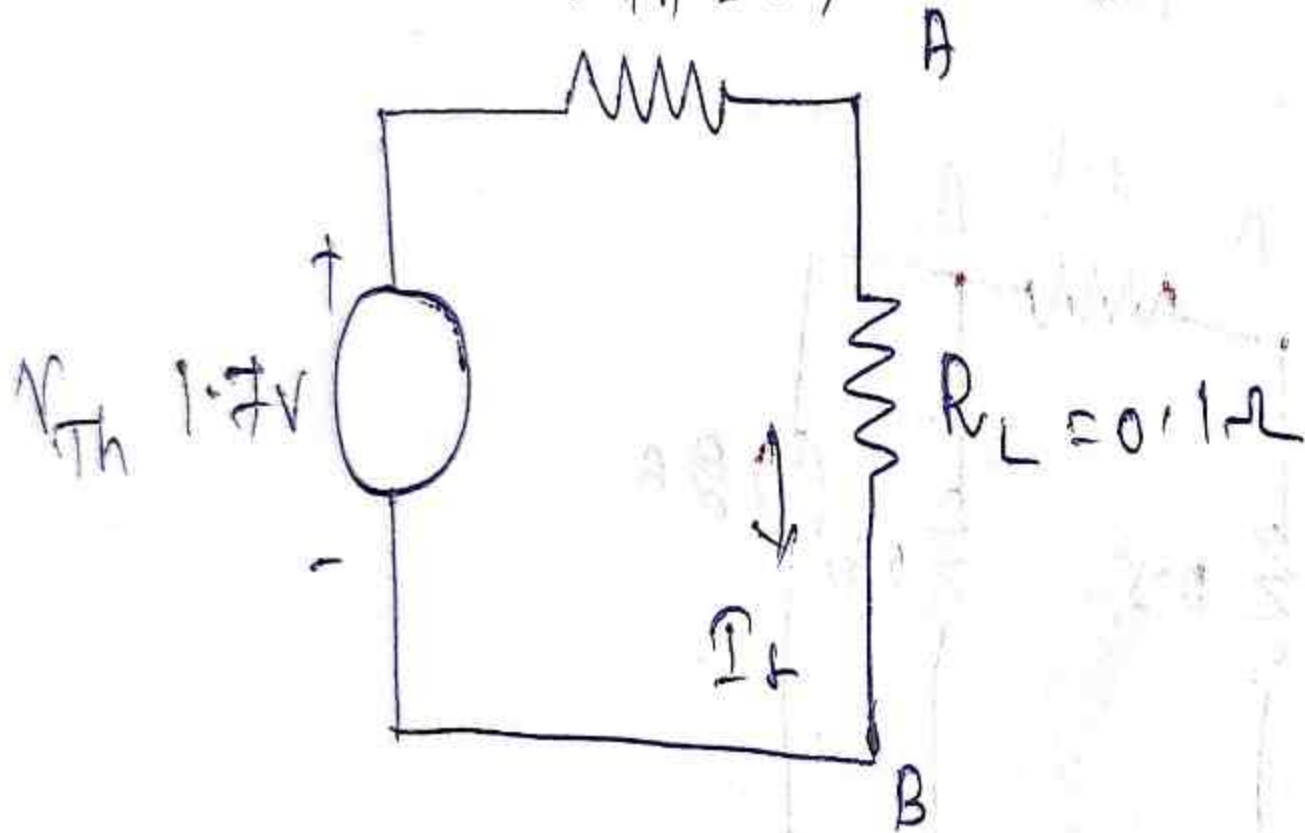
$$-V_{th} + 2.5 - 0.8 = 0$$

$$-V_{th} = -1.7V \Rightarrow V_{th} = 1.7V$$

(iii) $I_L = \frac{V_{th}}{R_{th} + R_L} = \frac{1.7V}{0.9 + 10} = 1.7A$

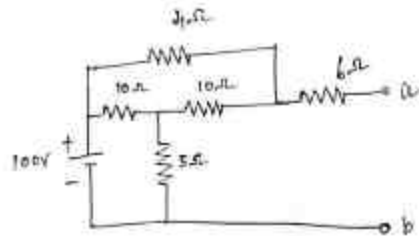
Equivalent circuit :

$$R_{Th} = 0.9 \Omega$$



Thevenin's Theorem Based problems:

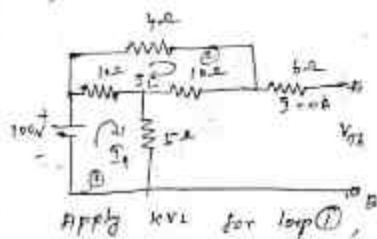
9) Find the Thevenin's Equivalent circuit at (a,b) (May 9, 16/2020)



Solution:

(i) V_{Th} , (ii) R_{Th}

(i) steps to find V_{Th} :
(Find the DC Voltage)



Apply KVL for loop ①,

$$-100 + 10(I_1 - I_2) + 5I_1 = 0$$

$$15I_1 - 10I_2 = 100 \rightarrow \text{①}$$

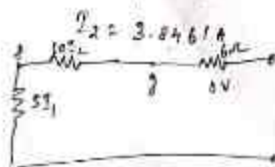
Apply KVL for loop ②,

$$4I_2 + 10I_2 + 8(10I_1 + I_2) = 0$$

$$-10I_1 + 24I_2 = 0$$

$$\Rightarrow 10I_1 - 24I_2 = 0 \rightarrow \text{②}$$

Solving, $I_1 = 9.2307A$



$$5 \times 2 = 10 \text{ } \rightarrow 10 \times 9.2307A$$

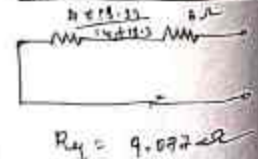
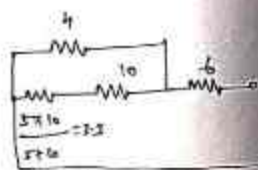
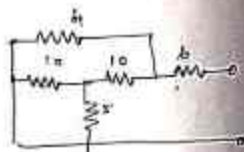
$$10 \times 2 = 20 \times 3.8461A$$

$$I_2 = 3.846A$$

$$V_{Th} = 30.44 + 3.846 \times 5 \Omega = 84.6145V$$

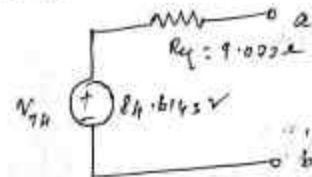
Step 2: R_{Th} :

Remove the sources

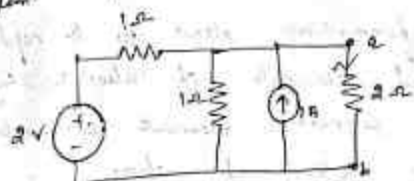


$$R_{Th} = 4.097 \Omega$$

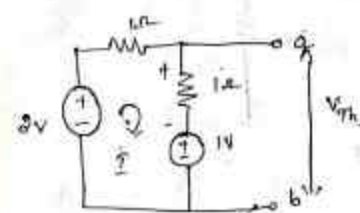
\therefore The Thevenin's equivalent circuit is,



10) Calculate the current through the 2Ω resistor in the circuit shown below using Thevenin's Theorem (Dec-10, May-12) (8 Marks)

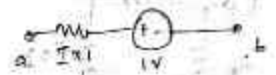


Step 1: For find V_{Th} (Remove or disconnect $R_L = 2\Omega$)



Apply KVL,

$$-2 + 2I + 1 = 0 \Rightarrow 2I = 1 \Rightarrow I = \frac{1}{2} = 0.5 \Rightarrow I = 0.5A$$



$$\therefore V_{Th} = 1 + 0.5 = 1.5V$$

$$\therefore V_{Th} = 1.5V$$

$$R_{Th} = \frac{1 \times 1}{1+1} = \frac{1}{2} = 0.5 \Omega$$

$$I = \frac{V_{Th}}{R_{Th} + R_L} = \frac{1.5}{0.5 + 2} = 0.6A \downarrow$$

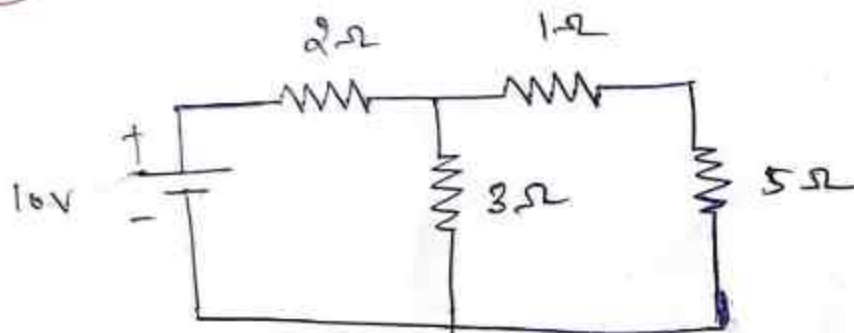
Theremin's Theorem Based

problem:

(16)

Find 'I' thru 5Ω

by Theremin's Theorem.



Hint:

For solving in Theremin's Theorem, we need to do following steps:

1. Remove ' R_L '

[R_L is nothing but the ' R ' I flowing
thru the particular ' R ' (which asked in
question)]

2. Apply KVL to find the loop currents.
3. Find V_{th} on the R_L part.
4. Find $R_{eq} \equiv R_{th}$ by removing all sources.
5. Draw Equi. circuit.
6. Find

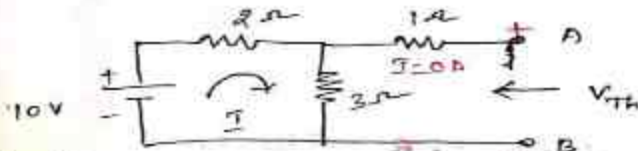
$$I_L = \frac{V_{th}}{R_{th} + R_L} \quad (R_{eq})$$



Solution:

Step 1:

Remove ' R_L ' to find V_{th} :



Step 2:
Apply

KVL to the loop:

$$-10 + 5I = 0 \Rightarrow 5I = 10 \Rightarrow \boxed{I = 2A}$$

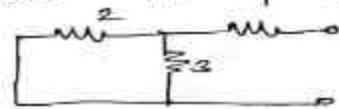
Step 3:

$$V_{th} = 3 \times I + 0 \times 1 = 3 \times 2 = \underline{\underline{6V}}$$

Step 4:

To find $R_{eq} (R_{th})$:

Remove the voltage source,

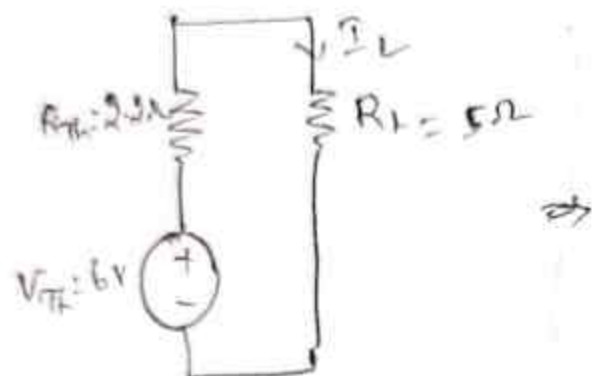


$$2 \parallel 3 \Rightarrow \frac{2 \times 3}{2 + 3} = \frac{6}{5} = 1.2 \Omega$$

1.2 ohm series with 1.

$$\therefore 2.2 \Omega \Rightarrow \boxed{R_{th} = 2.2 \Omega}$$

∴ step 5:
Circuit:



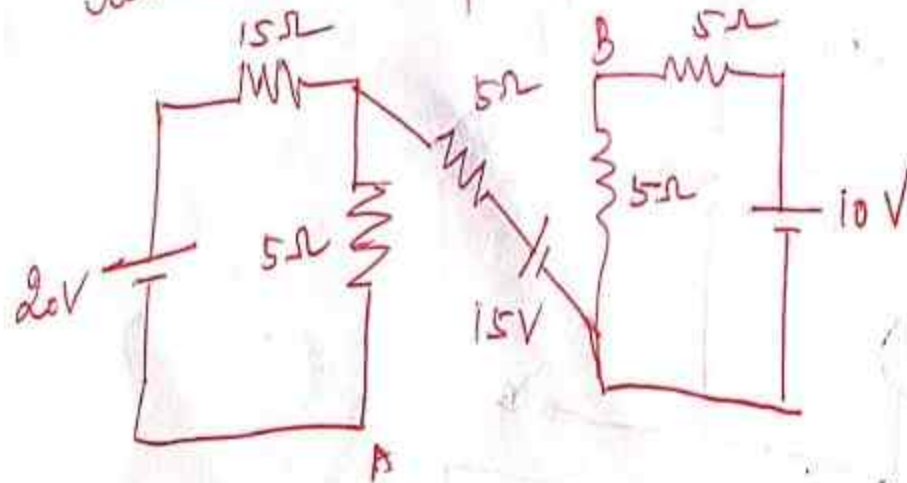
Step 6:

$$I_L = \frac{V_{Th}}{R_L + R_{Th}} = \frac{6}{2.2 + 5}$$
$$= \frac{6}{7.2}$$

$$I_L = 0.8333A$$

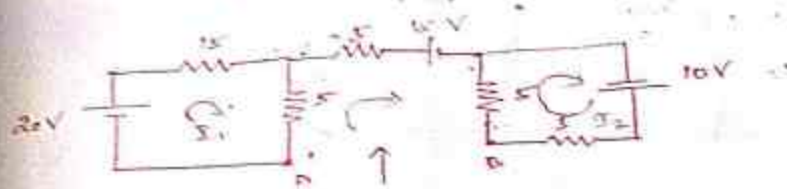
Thvenin's Theorem: Problems:

(18) Determine the Thvenin's equivalent across the terminals A & B as shown in figure.



Solution:

The given circuit can be redrawn as follows



Step 1: To find V_{th} :

~~Apply~~

Apply KVL to loop 1

$$-20 + 15I_1 + 5I_1 = 0$$

$$20I_1 = 20 \Rightarrow I_1 = 1A$$

Apply KVL to loop 2

$$-10 + 10I_2 = 0$$

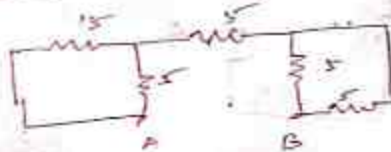
$$\Rightarrow I_2 = 1A$$

$$-5I_1 + 0 + 15 - 5I_2 = V_{th} = 0$$

$$-5 + 15 - 5 = -V_{th} = 0$$

$$V_{th} = -5V \Rightarrow V_{th} = 5V$$

Step 2: To find R_{th} :



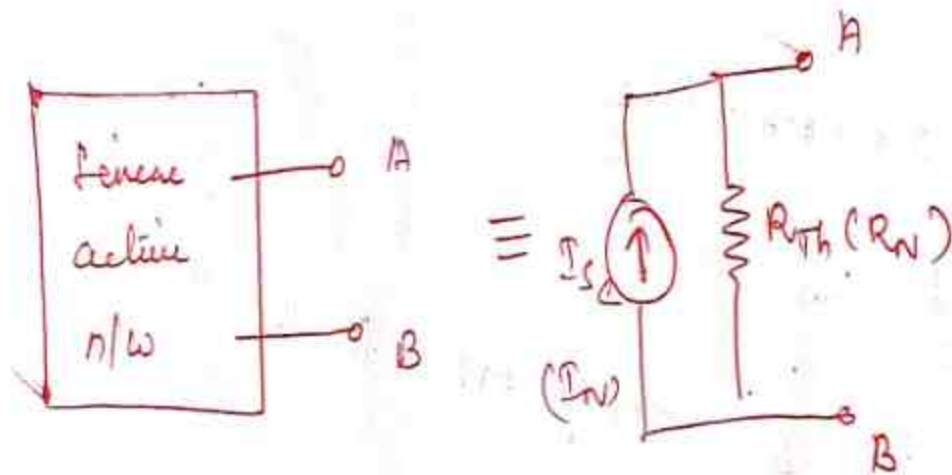
$$= \left(\frac{15 \times 5}{15 + 5} \right) + 5 + 5 \parallel 10 = \frac{75}{20} + 5 + \frac{25}{10} = 11.25 \Omega$$

Norton's Theorem:

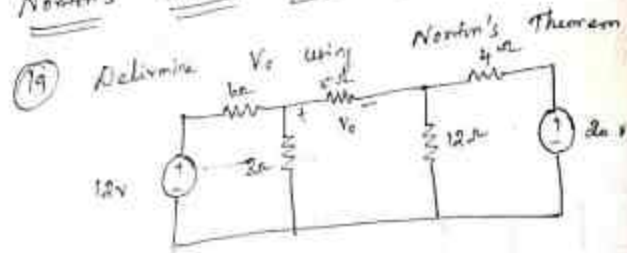
• Dual to Thevenin's Theorem

Statement:

"Any linear active n/w with open terminals A, B as shown in figure can be replaced by a single current source $I_{sc}(I_N)$ in parallel with a single impedance $Z_{Th} (= R_{Th} = R_N)$ "

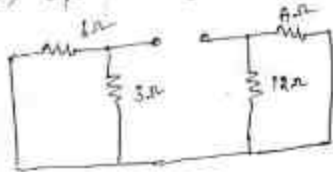


Norton's Theorem Based Problems



Step 1: $R_L = 5\Omega$ Remove R_L

Step 2: To find R_N :
 → Remove the R_L
 → Short voltage sources
 → Open current sources



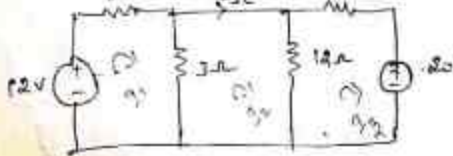
$$= \frac{6 \times 3}{6+3} + \frac{4 \times 12}{4+12}$$

$$= \frac{18}{9} + \frac{48}{16}$$

$$= 2 + 3 = 5\Omega$$

$$R_N = 5\Omega$$

Step 3: To find I_{sc} (or) I_N



$-2 + \frac{12 \times 12}{16} = 1.5A$

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$$\begin{bmatrix} 9 & -3 & 0 \\ -3 & 15 & -12 \\ 0 & -12 & 16 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ -20 \end{bmatrix}$$

$\therefore I_1 = 0.6A, I_2 = -2.2A, I_3 = -2.9A$

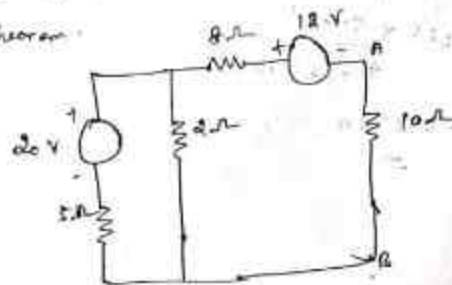
$$I_N = I_2 = -2.2A$$

$$I_1 = I_N \times \frac{R_N}{R_N + R_L} = 1.1A$$

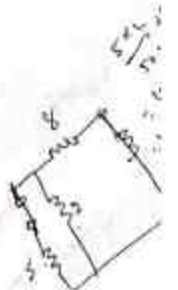
$$V_o = I_L R_L$$

$$V_o = 5.5V$$

20) Evaluate the I thro' 10Ω resistor by Norton Theorem.



Step 1: Remove R_L & short it.



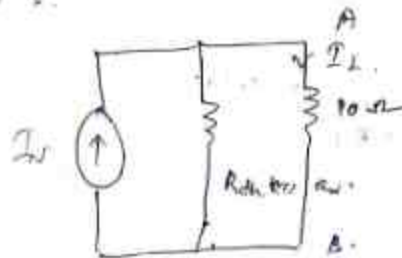
I_N :

$$\begin{bmatrix} 7 & -2 \\ -2 & 10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 20 \\ -12 \end{bmatrix}$$

$$I_2 = \frac{-44}{66} = -0.66$$

$$I_N = -0.66 \text{ A}$$

Step 2:



$$I_L = I_N \times \frac{R_N}{R_N + R_L}$$

$$= -0.666 \times \frac{9.428}{9.428 + 10}$$

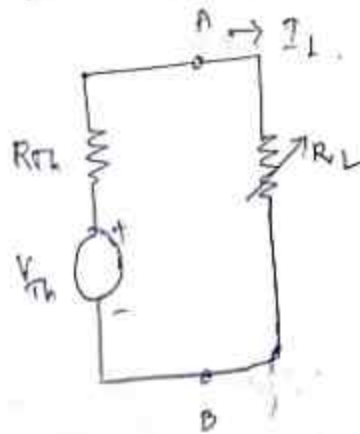
$$I_L = -0.328 \text{ A}$$

MAXIMUM POWER TRANSFER THEOREM:

if " Purely Resistive circuit & R_L is variable "

Statement :

" Maximum power will be delivered from a voltage source to a load, when the load R_L is equal to the internal R of the source "

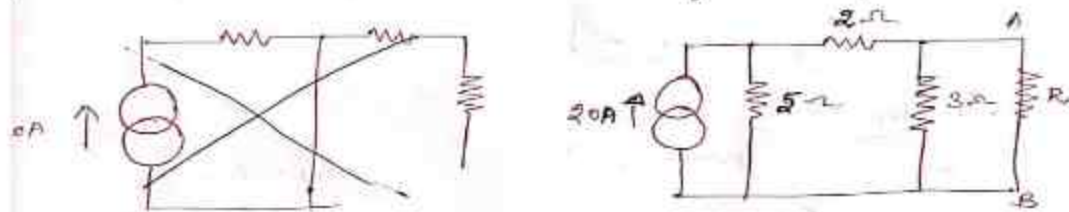


$$\therefore R_{Th} = R_L$$

$$\therefore I_L = \frac{V_{Th}}{R_L + R_L} = \frac{V_{Th}}{2R_L}$$

$$\begin{aligned}
 \text{Max. power delivered to } R_L &= \frac{I_{sc}^2 R_L}{4} \\
 &= \frac{V_{Th}^2}{4R_L} \times R_L \\
 &= \frac{V_{Th}^2}{4}
 \end{aligned}$$

F) The circuit shown in figure, R absorbs max. power. Compute the value of 'R' & max. power.

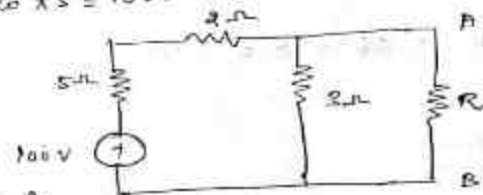


Solution:

Step 1:

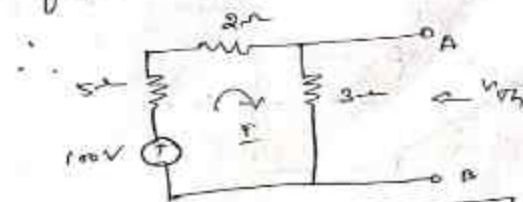
Convert current source into voltage source

$$20 \times 5 = 100V$$



Step 2:

To find V_{Th} Remove R_L (Here $R' = R_L$)

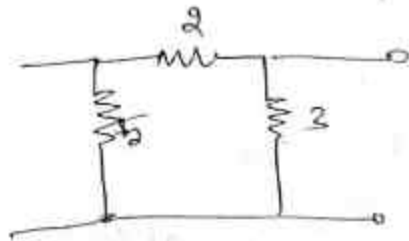


$$-100 + 10I = 0 \Rightarrow I = 10A$$

$$\therefore V_{Th} = V_{AB} = 3 \times I = 3 \times 10 = \underline{\underline{30V}}$$

Step 3: To find R_{th} :

Remove sources: & R_L



$$\therefore 2 \text{ \& } 3 \text{ in series} \Rightarrow 5 \parallel 5 = \frac{5 \times 5}{5 + 5} = \frac{25}{10} = 2.5 \Omega$$

$$\therefore R_{th} = 2.5 \Omega$$

Step 4:

At max. power, \mathcal{P}

$$R_{th} = R_L$$

$$\therefore I_L = \frac{V_{th}}{2 R_{th}} = \frac{V_{th}}{5} = \frac{30}{5} = 6 \text{ A}$$

$$P_L = \frac{V_{th}^2}{4 R_L} = \frac{30^2}{4 \times 2.5} = \frac{900}{10}$$

$$P_L = 90 \text{ W}$$

(or)

$$P_L = I_L^2 R_L = 36 \times 2.5 = 90 \text{ W}$$

($R_L = 2.5 \Omega$)

Dual Circuits

\Rightarrow "2 electrical n/w are said to be dual n/w if the mesh equations of one n/w is = to the node eqn of the other."

\Rightarrow In an electrical n/w, there is a pair of terms which can be interchanged to get new circuits called dual n/w.

Dual elements are:

Original Dual
 $I \leftrightarrow V$

I source \leftrightarrow V source

$R \leftrightarrow$ conductance (G)

$C \leftrightarrow L$

Loop \leftrightarrow Node

KCL \leftrightarrow KVL

series \leftrightarrow parallel
oc \leftrightarrow sc
bc \leftrightarrow followed

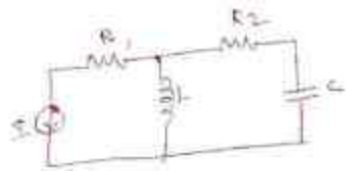
Steps to draw the dual n/w:
1. Impedance \leftrightarrow Admittance

① \Rightarrow In each loop, place node & place an extra node (Ref. node) outside the circuit.

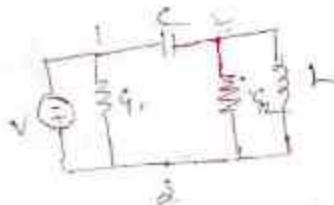
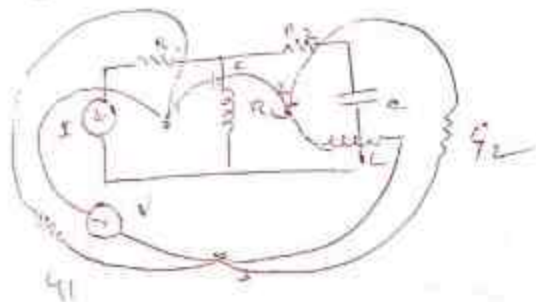
② \Rightarrow Draw the lines connecting adjacent nodes passing through each element and also to ref. node.

③ Draw dual n/w by replacing dual of each element in the line passing through original element.

Q1) Draw the dual n/w for the given n/w

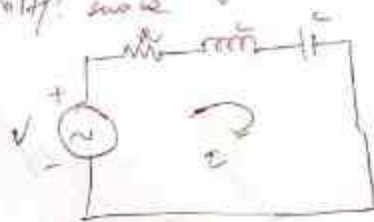


Soln:

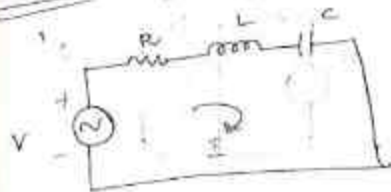


Q2) Consider R-L-C network circuit as shown

Verify source V



Solution:



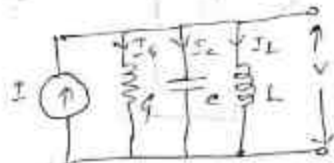
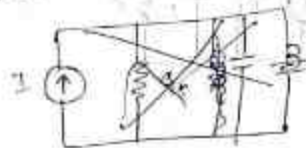
Apply KVL to this ckt,

$$-V + IR + L \frac{dI}{dt} + \frac{1}{C} \int I dt = 0$$

$$V = IR + L \frac{dI}{dt} + \frac{1}{C} \int I dt$$

$$V = V_R + V_L + V_C \rightarrow (1)$$

Draw:

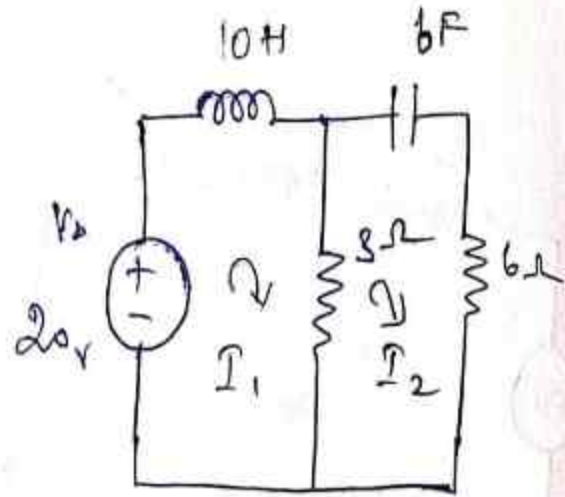
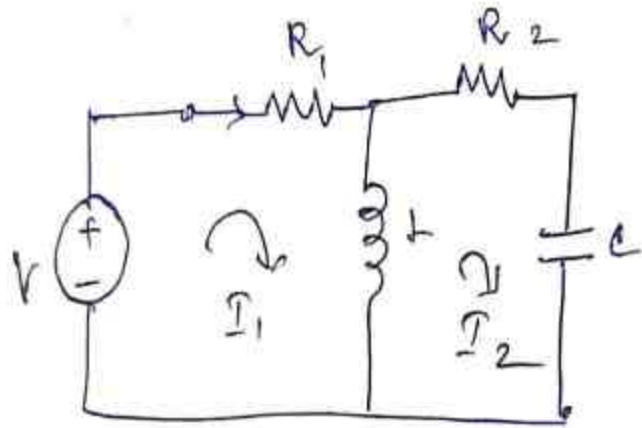


$$\Rightarrow I = I_R + I_C + I_L \rightarrow (2)$$

Conclusion:

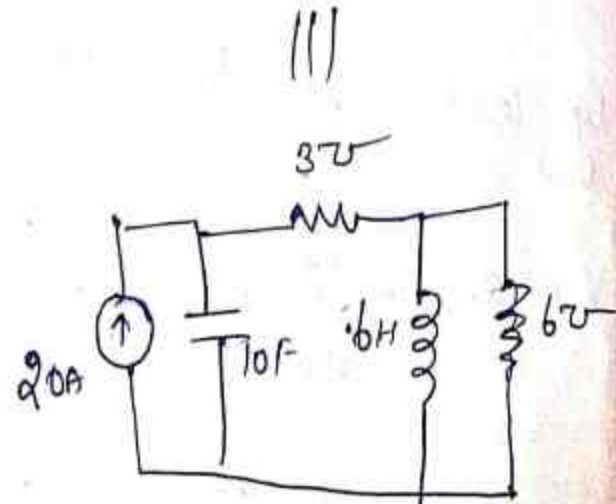
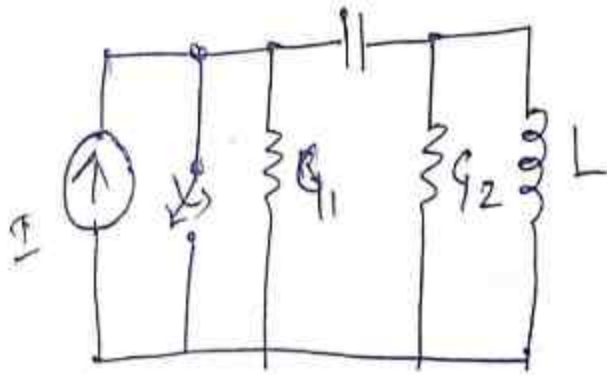
1. Eqn (1) & (2) are same
2. Mesh & node Eqn's are identical
3. They behave identically with interchanged variables

Q3 Draw the dual n/w for the given n/w.



Soln 1:

Dual n/w: c

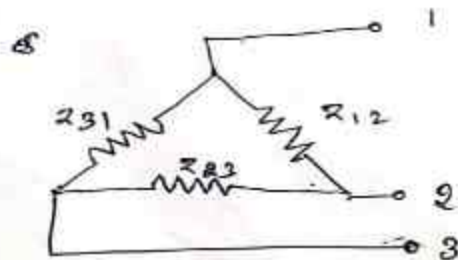
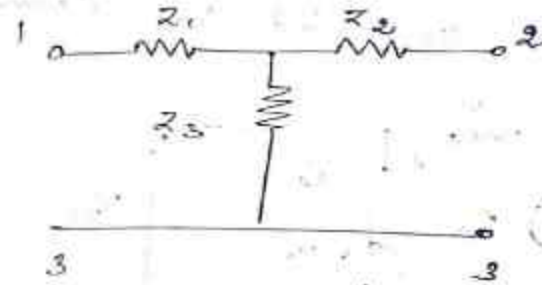
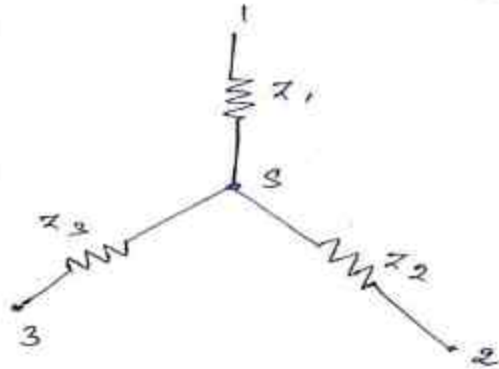


(T) (D) (T) (D)

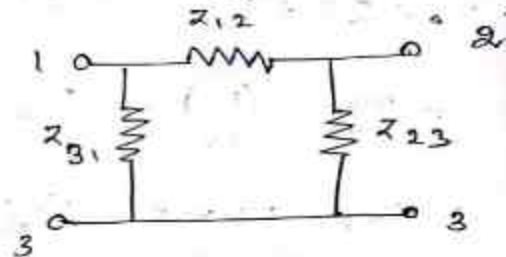
Delta - Star & Star - Delta Transformations:

⇒ For the complicated networks, these transformations considerably reduce the complexity of the network & analysed quickly.

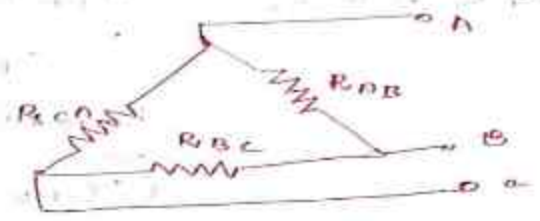
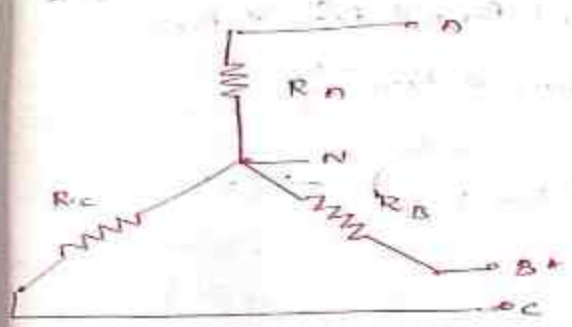
⇒ These transformations allow us to replace 3 star connected elements of network by equivalent delta connected elements without affecting 'I' in other branches.



Star (or) Γ connection



Star to Delta (or) π to Δ Transformations:



W.K.T,

$$R_A = \frac{R_{AB} R_{CA}}{R_{AB} + R_{BC} + R_{CA}} \rightarrow (1)$$

$$R_B = \frac{R_{BC} R_{AB}}{R_{AB} + R_{BC} + R_{CA}} \rightarrow (2)$$

$$R_C = \frac{R_{CA} R_{BC}}{R_{AB} + R_{BC} + R_{CA}} \rightarrow (3)$$

Multiply (1) & (2)

$$R_A R_B = \frac{R_{AB}^2 R_{BC} R_{CA}}{(R_{AB} + R_{BC} + R_{CA})^2} \rightarrow (4)$$

x (2) & (3)

$$R_B R_C = \frac{R_{BC}^2 R_{AB} R_{CA}}{(R_{AB} + R_{BC} + R_{CA})^2} \rightarrow (5)$$

x (3) & (1)

$$R_C R_A = \frac{R_{CA}^2 R_{AB} R_{BC}}{(R_{AB} + R_{BC} + R_{CA})^2} \rightarrow (6)$$

Add (A), (B) & (C), we get

$$R_A R_B + R_B R_C + R_C R_A = \frac{R_{AB} R_{BC} R_{CA} (R_{AB} + R_{BC} + R_{CA})}{(R_{AB} + R_{BC} + R_{CA})^2}$$

$$= \frac{R_{AB} R_{BC} R_{CA}}{(R_{AB} + R_{BC} + R_{CA})} \rightarrow (7)$$

$$= R_{AB} R_C$$

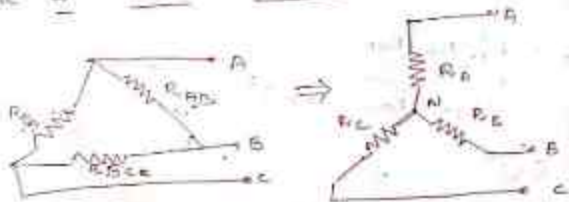
$$\Rightarrow R_{AB} = \frac{R_A R_B + R_B R_C + R_C R_A}{R_C}$$

Similarly, $R_{BC} = \frac{R_A R_B + R_B R_C + R_C R_A}{R_A}$

$$R_{CA} = \frac{R_A R_B + R_B R_C + R_C R_A}{R_B}$$

$$R_{AB}$$

Delta to Star Conversion



⇒ Easy to analyse

A, B, C → Terminals

⇒ To find eff. R^o w/o A & B (Delta connection)

$$R_{AB} = \frac{(R_{BC} + R_{CA}) R_{AB}}{R_{AB} + R_{BC} + R_{CA}} \rightarrow (8)$$

Star connection → Res. b/w A & B

$$R_{AB} = R_A + R_B \rightarrow (9)$$

$$(8) \equiv (9)$$

$$R_A + R_B = \frac{R_{AB} (R_{BC} + R_{CA})}{R_{AB} + R_{BC} + R_{CA}} \rightarrow (10)$$

Similarly,

$$R_{BC} = \frac{R_{BC} (R_{CA} + R_{AB})}{R_{AB} + R_{BC} + R_{CA}} \rightarrow (11)$$

∴

$$R_B + R_C = \frac{R_{BC} (R_{CA} + R_{AB})}{R_{AB} + R_{BC} + R_{CA}} \rightarrow (12)$$

Similarly,

$$R_C + R_A = \frac{R_{CA} (R_{AB} + R_{BC})}{R_{AB} + R_{BC} + R_{CA}} \rightarrow (13)$$

$$(10) - (12) \Rightarrow R_A - R_C = \frac{R_{AB} R_{BC} + R_{AB} R_{CA} - R_{BC} R_{CA} + R_{BC} R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$$

$$R_A - R_C = \frac{R_{AB} R_{CA} - R_{BC} R_{CA}}{R_{AB} + R_{BC} + R_{CA}} \rightarrow (14)$$

Add (8) & (9) ⇒

$$2 R_A = \frac{R_{AB} R_{CA} + R_{CA} R_{BC} + R_{AB} R_{CA} - R_{BC} R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$$

$$R_A = \frac{R_{AB} R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$$

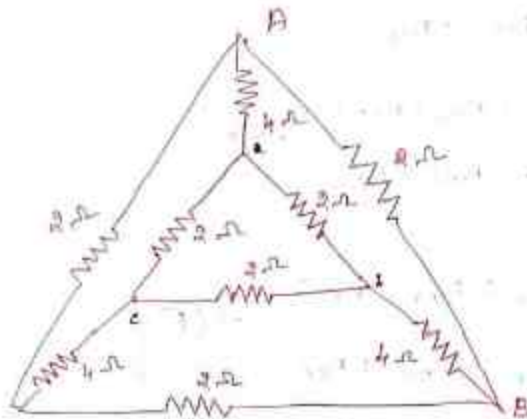
$$R_A = \frac{R_{AB} R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$$

Key,

$$R_B = \frac{R_{BC} R_{AB}}{R_{AB} + R_{BC} + R_{CA}}$$

$$R_C = \frac{R_{CA} R_{BC}}{R_{AB} + R_{BC} + R_{CA}}$$

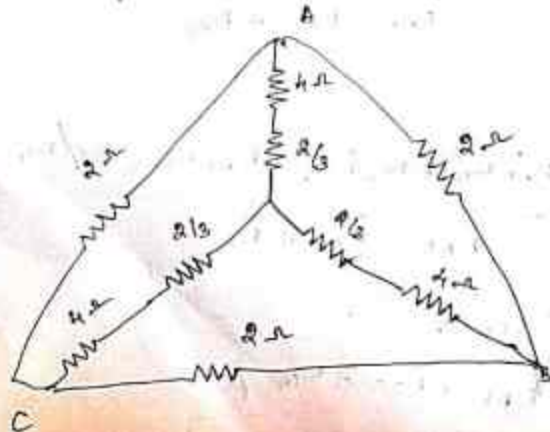
① Find the Req. b/w A & B for the given circuit



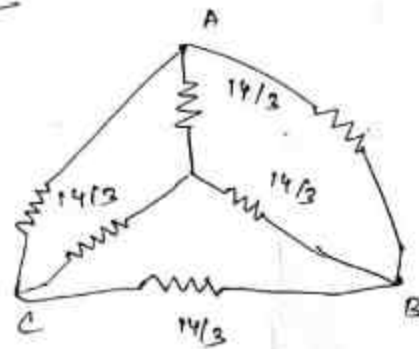
$2, 2, 2 \Rightarrow$ in center we in Δ or Π

Step 1:

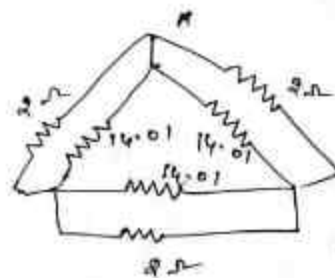
Δ to Star



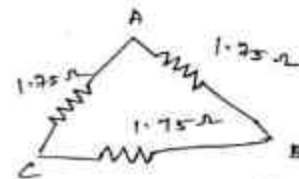
Step 2:



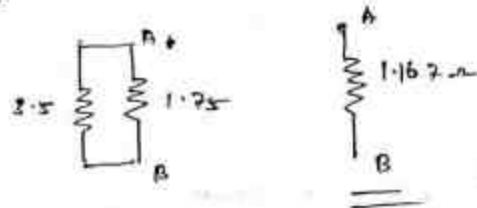
Step 3:



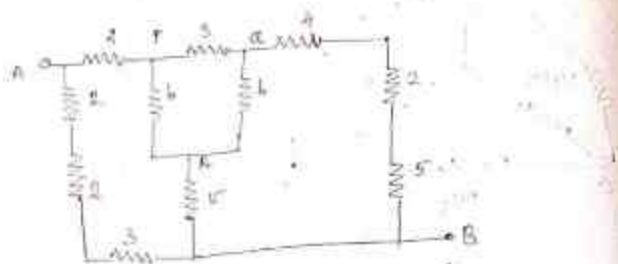
Step 4:



Step 5:

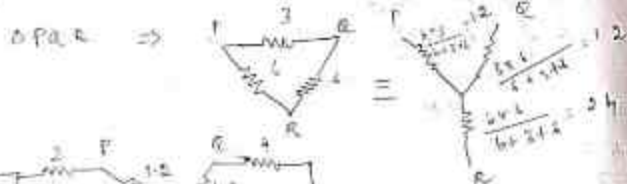


12. Calculate the effective R b/w points A & B in the given circuit.



Solution:

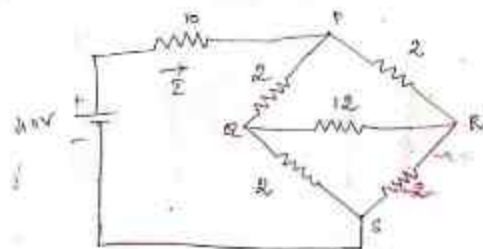
step 1:



$$\frac{1}{R_{eq}} = \frac{1}{1.2} + \frac{1}{2.4} + \frac{1}{2} = 0.3064$$

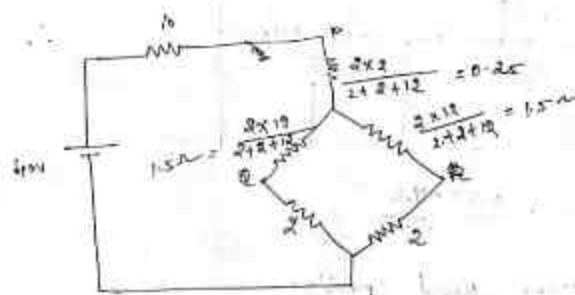
$$\therefore R_{eq} = 3.262$$

13. For the circuit shown in figure, find the current flowing through the 10Ω R.



Solution:

Step 1: Apply Δ to T \Rightarrow in PAR



Step 2:



Step 3:

$$2.5 \parallel 2.5 \Rightarrow 1.25 \text{ series with } 0.2\Omega$$

$$I = \frac{40}{10 + 0.2 + 1.25}$$

$$I = 3.33A$$